Learning Goals

1. Rankings vs. ratings.
2. Definition of Condorcet winner and Weak Condorcet winner.
3. Concept of a social choice function.
4. Plurality method.
5. Problems with the plurality method.
6. Preference ranking.
7. Runoff Voting.
8. Instant runoff voting.

Topic 5 : Choosing a winner for a Round Robin and Preference Rankings

Point Differential and Wins - Losses

If we run a round robin tournament we must decide on who gets first and second place, or as fans we often want to decide who is number one half way through the tournament. We will see that the team or person who wins these honors may depend heavily on the method we use to decide on the winner.

Example The table on the left below shows partial results of a round robin; the 2015 Six Nations Championship in Rugby (up to Feb. 25 2015) and the one on the right shows the final results. In February, the rugby world is concerned with ranking the teams and in particular making a declaration as to who is number 1. When the tournament is over, a decision as to who actually wins the cup must be made. It is of course best to clarify how this decision will be made before the tournament begins, since there are several ways to do so and different methods may lead to different winners.

Feb 25	Ire.	Eng.	Wal.	Scot.	Fra.	It.
Ire.					18-11	26-3
Eng.			21-16			47-17
Wal.		16-21		26-23		
Scot.			23-26		8-15	
Fra.	11-18			15-8		
It.	3-26	17-47				

Full	Ire.	Eng.	Wal.	Scot.	Fra.	It.
Ire.		19-9	16-23	40-10	18-11	26-3
Eng.	9-19		21-16	25-13	55-35	47-17
Wal.	23-16	16-21		26-23	20-13	61-20
Scot.	10-40	13-25	23-26		8-15	19-22
Fra.	11-18	35-55	13-20	15-8		29-0
It.	3-26	17-47	20-61	22-19	0-29	

A **Ranking** refers to a rank ordered list of the competitors and a **Rating** gives us a list of numerical scores for the competitors. Every rating gives us a ranking of the competitors.

dated Example from 2017 If we wanted to decide who was the best basketball player among the three, Stephen Curry, LeBron James and Kevin Durant, we might search around the web for rankings some of which (from 2017) are shown in the table on the left below:

	Off. Ranking UPROXX	Sports Ill.	Was. Post	Fantasy BB Power R
Kevin Durant	1	2	3	1
LeBron James	2	1	1	3
Stephen Curry	3	3	2	2

	FG%	+/-	OE
Kevin Durant	53.7	10.4	0.5897
LeBron James	50.8	10.2	0.6094
Stephen Curry	46.5	12.3	0.5528

On the other hand we might go to `nba.com` to find some current statistics which will give us a rating for each player or we might calculate our statistic OE from the last section. The stats are shown in the table on the right above. The ratings on the right can be converted to the rankings shown below:

	FG%	+/-	OE
Kevin Durant	1	2	2
LeBron James	2	3	1
Stephen Curry	3	1	3

Two commonly used statistics to assign ratings to competitors in a round robin are **Wins minus Losses** (W-L) and **Point Differential** (PD). The major drawbacks to W-L is that it often leads to ties and one needs a method of breaking ties. In addition, it does not take into account the magnitude of a win or a loss. On the other hand if one uses the point differential to rank teams, stronger teams may deliberately run up the score against weaker teams to improve their rankings thus distorting the results unless the strength of the opponent is taken into account. You will see later that both of these statistics are used as a basis for popular computer methods used to determine rankings for teams.

Example (a) **Partial Results** Lets look at both of these statistics for the results of the partial (Feb. 25) results of the 2015 Six Nations Rigby Cup. When counting Wins and losses, we count a tie as a half win and a half loss. Of course the statistics W-L and PD will have the same value whether we ignore ties or count them as above. In this case there are no ties to count.

Player	**Name**	**W = Wins**	**L = Losses**	**W-L**	**PD**
1	Ireland	2	0		
2	England	2	0		
3	Wales	1	1		
4	Scotland	0	2		
5	France	1	1		
6	Italy	0	2		

Part way through the tournament, statistics are generally used to rank the teams and try to make predictions about who will win. In this case if we decide on a ranking based on W-L alone, we have a problem with ties. We can however break the ties with the PD statistic. Notice that in this case W-L (with ties broken by the PD) gives the same ranking as the ranking we would get if we just used the PD to rank the teams. This is not always the case.

(b) **Deciding on a winner**

Player	**Name**	**W = Wins**	**L = Losses**	**W-L**	**PD**
1	Ireland	4	1		
2	England	4	1		
3	Wales	4	1		
4	Scotland	0	5		
5	France	2	3		
6	Italy	1	4		

At the end of the tournament, we need to decide on a ranking to determine who is in first, second third and fourth place etc.... In this case, we have a problem with ties again if we try to use W-L to determine the winner, but we can break the tie with the point differential. It is interesting to note here that if we use this method, we get a different ranking for the fourth fifth and sixth places than if we used the point differential alone.

A Condorcet winner in a round robin tournament is a competitor who wins against every other competitor. If such a competitor exists, they will be the unique competitor with such a property, however, they do not always exist as is the case in the above tournament. A **Weak Condorcet Winner** is a competitor who wins or draws against all of the other competitors. There may be more than one weak Condorcet winner or there may be none as is the case in the example above.

A Social Choice function

We see that deciding on a winner requires us to choose a method of rankling competitors and quite often different methods lead to different winners. Different statistics often give different rankings and opinions of judges often disagree. Mathematicians and economists have struggled with **the problem of finding a fair way to amalgamate different rankings into a single ranking (or social choice function) to produce a winner.** Clearly an amalgamation of rankings is essential in sports where one takes into account several different statistics or the opinion of several judges. The use of a social choice function is also essential in a democracy, where candidates are ranked by voters and the votes must be amalgamated to choose a winner of an election. Much of the theory has been developed in the context of politics and in the discussion of various methods of amalgamating votes below, we will refer to candidates (which you can think of as competitors in sports) and voters (which you can think of as judges or statistics) who fill out a ballot showing how they rank the candidates.

Plurality and Runoff Methods for choosing a winner

Plurality Method

One very simple method of Voting is
The Plurality Method With this method, each voter selects one candidate or choice on the ballot. The winner is the candidate or choice with the most votes.

Example 1 A committee of 10 people (with names A, B, . . . , J) must vote on a venue for their next Gymnastics competition. The choices are Indianapolis, South Bend, Fort Wayne, Terre Haute. The committee uses the plurality method of voting, and their ballots are given in the following table:

	A	B	C	D	E	F	G	H	I	J
Indianapolis		X				X	X			X
South Bend	X			X				X		
Fort Wayne									X	
Terre Haute			X		X					

Which venue did they choose?

Problems With The Plurality Method

Sometimes this method leads to a **tie**. This is less likely when there are large numbers of voters.

Example Let us put together our rankings for the three basketball players from our example above and use the method of plurality to decide who is Number 1.

	Off. Ranking UPROXX	Sports Ill.	Was. Post	Fantasy BB Power R	FG%	=/-	OE
Kevin Durant	1	2	3	1	1	2	2
LeBron James	2	1	1	3	2	3	1
Stephen Curry	3	3	2	2	3	1	3

We see that Kevin Durant got 3 Number 1's, LeBron James got 3 Number 1's, and Stephen Curry got one Number 1. Thus we have a tie for first place and we must have some tie breaking rules in place to deal with such a situation.

A more serious problem with plurality voting is that it leads to a **Splitting of the vote on similar choices;** If there are just two choices or candidates and the plurality method is used, then the popular choice is guaranteed to win. However if there are more than two choices then it may happen that more extreme choices may win over many similar, but popular choices.

Example 2 Suppose a group of 10 people, many of whom like camping and hiking activities are deciding on where to spend fall break. The options are Camping and Hiking in Colorado, Camping and Hiking in California, Camping and Hiking in Washington, Camping and Hiking in Ireland, Disneyworld. Using the Plurality method the group may end up with a vote like this

	#Voters
Camping and Hiking in Colorado	2
Camping and Hiking in California	2
Camping and Hiking in Washington	2
Camping and Hiking in Ireland	1
Disneyworld	3

Clearly Camping and Hiking is preferred to Disneyworld, but beacause there are so many similar options for Camping and Hiking, the group ends up going to Disneyworld.

Another problem with the plurality method is that there may be an incentive for **Strategic Voting**. Voters supporting a weak choice may be inclined to change their vote to vote strategically.

Strategic voting occurs when a voter votes in a way that does not reflect their true preferences in an attempt to improve the outcome of the election/poll.

In Example 1 above, suppose that the voters who prefer Terre Haute know that nobody else will vote for Terre Haute. Suppose also that these voters prefer South Bend to Indianapolis, how can they improve the chances that South Bend will win?

	A	B	C	D	E	F	G	H	I	J
Indianapolis		X				X	X			X
South Bend	X			X				X		
Fort Wayne									X	
Terre Haute			X		X					

Runoff Voting

Because of the problems with plurality method, a runoff election is often used.

In a **Runoff Election**, a plurality vote is taken first.

1. If one of the candidates has more than 50% of the votes, that candidate wins.

2. If no candidate has has more than 50% of the votes, a second round of plurality voting occurs with a designated number of the top candidates.

3. The process continues until one of the candidates has more than 50% of the votes.

Example: Olympic Voting

The selection of the site for the Olympic Games is made by the International Olympic Committee. The voting process calls for a plurality election with a runoff between all of the candidates except the one in the last place. (This is known as the Hare Method). A number of controversial results have led to suspicions about strategic voting in the past. The results of the election for the location of the 2016 summer olympics are shown below.

Election of the Host City of the 2016 Summer Olympics — ballot results				
Candidate City	**Country (NOC)**	**Round 1**	**Round 2**	**Round 3**
Rio de Janeiro	Brazil (BRA)	26 (27.66%)	**46** (48.42%)	**66** (67.35%)
Madrid	Spain (ESP)	**28** (29.79%)	29 (30.53%)	32 (32.65%)
Tokyo	Japan (JPN)	22 (23.40%)	20 (21.05%)	—
Chicago	United States (USA)	18 (19.15%)	—	—
121st IOC Session	**Vote details**	**Round 1**	**Round 2**	**Round 3**
Copenhagen - Denmark	Eligible	95	97	99
	Participants	94	96	98
	Abstentions	0	1	0
	Valid ballots	94	95	98
Members unable to vote				
Members from countries with candidate cities		Other members		
Anita L. Defrantz · James L. Easton · Chiharu Igaya · Shun-Ichiro Okano · João Havelange · Carlos Arthur Nuzman · Juan Antonio Samaranch Jr.		Jacques Rogge (IOC president) · Lee Kun-hee (suspended) · Alpha Ibrahim Diallo (absent) · Saku Koivu (absent)		

Can you find evidence of strategic voting (Hint: the number of votes for any particular city should not drop from one round to the next)?

Preference Ranking

In most voting situations, each voter has an order of preference of the candidates. Such an ordering is called a **Preference Ranking**. The voter may have to put some thought into making such a preference ranking and it may change over time.

The voting systems discussed below which use preference rankings make the following assumptions about them:
1. Each voter has a preference ranking that orders all candidates from most preferred to least preferred. (we assume that in the case of indifference or lack of knowledge of the candidates, the voter will choose a ranking randomly).
2. If a voter has ranked one candidate higher than another, then if the voter must choose between the two candidates, the voter would choose the higher ranked one.
3. The order of the preferences is not changed by the elimination of one or more candidates.

Note that this allows us to conduct a runoff election without revoting and it does not allow strategic voting where candidates change their preferences after the first round.

Example A fourth grade class is asked to rank their preferences for a field trip to a game of football basketball or baseball. The preference rankings of the voters are presented in a table below showing the number of voters with each preference in the top row.

# voters	**1**	**3**	**3**	**2**	**4**	**5**
football	1	1	3	2	4	3
basketball	2	4	1	4	1	4
baseball	3	2	4	3	2	1
soccer	4	3	2	1	3	2

(a) In a plurality election, which option would win?

(b) In a plurality election with a runoff between the top two finishers, what would the outcome be?

Instant Runoff Voting (IRV) (used in deciding winner of Oscars)

In an Instant Runoff election,
1. each voter ranks the list of candidates in order of preference. The candidates are ranked in ascending order with a "1" next to the most preferred candidate, a "2" next to the second most preferred candidate and so forth.
(In some implementations, the voter ranks as many or as few choices as they wish while in others they are required to rank all of the candidates or a prescribed number of them.)
2. In the initial count, the first preference of each voter is counted and used to order the candidates. Each first preference counts as one vote for the appropriate candidate.
3. Once all the first preferences are counted, if one candidate holds a majority (more than 50% of votes cast), that candidate wins. Otherwise the candidate who holds the fewest first preferences is eliminated.
(If there is an exact tie for last place in numbers of votes, tie-breaking rules determine which candidate to eliminate.)
4. Ballots assigned to eliminated candidates are recounted and assigned to one of the remaining candidates based on the next preference on each ballot.
5. The process repeats until one candidate achieves a majority (more than 50%) of votes cast for continuing candidates. Ballots that 'exhaust' all their preferences (all its ranked candidates are eliminated) are set aside.

Example In an instant runoff election which of the candidates in the previous example would win?

 # 1 votes

# voters	**1**	**3**	**3**	**2**	**4**	**5**	**R1**	**R2**	**R3**	**R4**
football	1	1	3	2	4	3				
basketball	2	4	1	4	1	4				
baseball	3	2	4	3	2	1				
soccer	4	3	2	1	3	2				

Example In an instant runoff for the three basketball players from our example above who is Number 1?

	Off. Ranking UPROXX	Sports Ill.	Was. Post	Fantasy BB Power R	FG%	=/-	OE
Kevin Durant	1	2	3	1	1	2	2
LeBron James	2	1	1	3	2	3	1
Stephen Curry	3	3	2	2	3	1	3